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Modelling non-stationary variance in EEG time series by state space GARCH model

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Running head: State space GARCH Model on EEG

Abstract

We present a new approach to modelling non-stationarity in EEG time series by a generalized state space approach. A given time series can be decomposed into a set of noise-driven processes, each corresponding to a different frequency band. Non-stationarity is modelled by allowing the variances of the driving noises to change with time, depending on the state prediction error within the state space model. The method is illustrated by an application to EEG data recorded during the onset of anaesthesia.

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1 Introduction

Brain dynamics can be analyzed by estimating the frequency spectrum of the electroencephalogram (EEG), which can be done by parametric or non-parametric methods; Fast Fourier Transform (FFT) represents a well-known nonparametric method (Priestley [1], Kay [2]), while fitting of autoregressive (AR) models is a prominent example of a parametric method (Box and Jenkins [3], Gersch [4]). However, in the case of the presence of pronounced non-stationarity in the EEG, such as time-dependent changes of the power in different frequency bands, direct application of the FFT to the data would be inappropriate.

Although in this case it is still possible to apply the FFT to a window moving over the data, such approach would have the disadvantage of reduced resolution either in time or in frequency domain; improved resolution in time domain, desirable in order to pick out distinctive temporal characteristics in the data, has to be paid by reduced resolution in frequency domain, and vice versa.

In contrast, parametric spectral estimation by AR models offers various advantages over the FFT, since it represents a more general and flexible framework for parsimonious dynamical modelling of time series data, which can be readily employed for purposes such as prediction, classification or causality analysis of time series (Shumway and Stoffer [5]); in the case of non-stationarity, parametric spectral estimation may also be applied to a moving window (Ozaki and Tong [6]), but as we will show in this paper, there is an alternative approach for this situation which avoids the introduction of a moving window.

We will model the EEG by a linear autoregressive model in a state space framework, which is suitable for describing the simultaneous presence of several major frequency bands; the non-stationarity will be added by employing the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, as introduced by Engle [7] for the modelling of time-dependent variance, and generalized by Bollerslev [8], but in contrast to the usual usage of the GARCH model we will apply it within state space.

The concept of employing the GARCH model within a state space has been introduced by Galka *et al* [9] in a study on the estimation of inverse solutions from EEG time series; for simulated data they obtained improved reconstruction of true states by this technique.

It is a property of the GARCH model that non-stationarities can be detected with very good temporal resolution (Bollerslev [8]); therefore we propose it as an appropriate tool for the modelling of transients and rapid changes of spectral properties, as they are commonly observed in EEG time series. As an example we will discuss in this paper the case of a clinical EEG time series displaying the transition from awake conscious state to anaesthesia.

2 An example serving as motivation

The EEG time series which we will study in this paper was retained from a recent study of John *et al* [10] and John [11] who have studied the change of spectral content of clinical EEG accompanying the loss and subsequent recovery of consciousness due to initiation and termination of anaesthesia during surgery. The data was measured at 19 electrodes fixed to the scalp according to the international 10/20 System. The detailed experimental procedures have been described in John *et al* [10] and Prichep *et al* [12].

Based on techniques from descriptive statistics, including FFT spectrum estimation and computation of mean z-scores, John *et al* [10] found that an increase in absolute power of the low frequency band occurs when patients lose consciousness. In this paper we will study the same topic and confirm their result by a parametric approach.

We select from their data a segment of 2048 samples from the T4 electrode (versus average reference), sampled at 100 Hz, such that the segment extends over about 20 seconds. This data set covers the transition from awake conscious state to anaesthesia. The data is shown in Figure 1.

——— Figure 1 about here ———

At time 0 seconds, induction of anaesthesia begins. At about 10 seconds, loss of consciousness occurs. It can be seen in the figure that starting from this time the amplitude of low-frequency activity in the EEG displays a pronounced increase.

In the lower panel of Figure 1 the spectral density according to a "conventional" moving window analysis is shown: The data was divided into 15 segments, each of length 256 samples, where consecutive segments overlap by 128 samples. Autoregressive models of order 8 are fitted to each segment, and the resulting parametric spectra are displayed. A peak at 10 Hz can be seen persistently; at low frequency the power rises at about 10 seconds.

3 Methods

The EEG data as shown in Figure 1 appears to be a superposition of several different source components. We intend to decompose the data into source components corresponding to several major spectral bands. Each component is described by a dynamical process, and each of these processes may have a different non-stationary behaviour of variance.

Let y_t denote the observed data and x_t the unobserved state. We assume that x_t depends on its past values through a linear stochastic model, containing a dynamical noise term, and that y_t follows from x_t through a linear observation model, containing an observation noise term; then the following state space model applies:

$$x_t = F x_{t-1} + w_t \tag{1}$$

$$y_t = H x_t + \epsilon_t \tag{2}$$

Equations (1) and (2) are commonly known as system equation and observation equation, respectively. w_t denotes the dynamical noise term of the system equation, assumed to follow a multivariate Gaussian distribution $w_t \sim N(0, Q_t)$, while ϵ_t denotes the observation noise term of the observation equation, assumed to follow a univariate Gaussian distribution $\epsilon_t \sim N(0, R)$.

Kalman [13] introduced a filtering technique for state space models which can efficiently calculate the conditional prediction and conditional filtered estimation of unobserved states. A comprehensive introduction to state space models and Kalman filtering has been provided by Kalman [13], Harrison and Stevens [14], Harvey [15], Grewal and Andrews [16].

Since we aim at decomposing the data into a set of source components, we choose a special structure for the state space model, such that pairs of elements within the state vector x_t represents autoregressive models of second order, AR(2). Each AR(2) model is capable of describing one main frequency found in the data. For a detailed account of the representation of oscillations by AR(2) models see Box and Jenkins [3].

Assume that r denotes the number of source components, then the state x_t is a column vector of dimension $2r$, F is a $2r \times 2r$ matrix and H is a $2r$

row vector. For each of the r source components there is a 2×2 block on the diagonal of F, all other elements of F are zero. Each 2×2 block contains two parameters, which define frequency and damping of the corresponding AR(2) model; thereby each component contributes to modelling the spectrum of the data.

The state vector and the two parameter matrices of the model then become

$$x_t = \begin{bmatrix} z_t^{(1)} \\ z_{t-1}^{(1)} \\ z_t^{(2)} \\ z_{t-1}^{(2)} \\ \vdots \\ z_t^{(r)} \\ z_{t-1}^{(r)} \end{bmatrix}, \quad F = \begin{bmatrix} \phi_1^{(1)} & \phi_2^{(1)} & 0 & \cdots & \cdots & \cdots & 0 \\ 1 & 0 & 0 & \ddots & & & \vdots \\ 0 & 0 & \phi_1^{(2)} & \phi_2^{(2)} & \ddots & & \vdots \\ \vdots & \ddots & 1 & 0 & 0 & \ddots & \vdots \\ \vdots & & \ddots & 0 & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \ddots & \phi_1^{(r)} & \phi_2^{(r)} \\ 0 & \cdots & \cdots & \cdots & 0 & 1 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} c^{(1)} & 0 & c^{(2)} & 0 & \cdots & c^{(r)} & 0 \end{bmatrix}$$

Q_t denotes the variance matrix of the dynamical noise w_t; also this matrix is composed of 2×2 blocks on the diagonal, each block corresponding to one AR(2) model, but here only one element within each block is nonzero (Harvey 1989):

$$Q_t = \begin{bmatrix} \left(\sigma_t^{(1)}\right)^2 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \ddots & & & & \vdots \\ \vdots & \ddots & \left(\sigma_t^{(2)}\right)^2 & 0 & & & \vdots \\ \vdots & & 0 & 0 & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \left(\sigma_t^{(r)}\right)^2 & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \end{bmatrix}$$

The nonzero elements $\left(\sigma_t^{(k)}\right)^2$ are time-dependent; according to the GARCH approach in its general form, they are modelled by

$$\log\left(\sigma_t^{(k)}\right)^2 = \log\left(\sigma_0^{(k)}\right)^2 + \sum_{i=1}^{p} \alpha_i^{(k)} \log\left(\sigma_{t-i}^{(k)}\right)^2 + \sum_{j=1}^{q} \beta_j^{(k)} \log\left(v_{t-j}^{(k)}\right)^2 \qquad k = 1, 2, \ldots, r \tag{3}$$

The first term on the right-hand side of this equation, $\log\left(\sigma_0^{(k)}\right)^2$ represents a constant, which in general may be non-zero, but here we will set it zero in order to avoid parameter redundancy. The first sum represents an autoregressive model of order p for the logarithm of the variance, while the second sum corresponds to a moving-average (MA) dynamical noise term. Expressing this GARCH model by the logarithm of the variance has the beneficial effect of preventing the variance from becoming negative.

In the original GARCH model the dynamical noise v_{t-j} would be given by previous values of the prediction error of the data, thereby feeding back increases of the prediction error into the variance of the dynamical noise; but in the state space approach the state prediction errors are not directly accessible, therefore we estimate it by forming the matrix $\mathrm{E}\left(w_{t-j}w'_{t-j}|y_{t-j}\right)$ (where E(.) denotes expected value) and extract from its diagonal the first element for $\log\left(\sigma_t^{(1)}\right)^2$, the third element for $\log\left(\sigma_t^{(2)}\right)^2$, etc. The detailed derivation and resulting expression for $\mathrm{E}\left(w_{t-j}w'_{t-j}|y_{t-j}\right)$ can be found in the Appendix.

The model parameters in Equations (1), (2) and (3) are estimated from given data by the maximum-likelihood method. Given a set of parameters, computation of the likelihood from the errors of the data prediction through application of the Kalman filter is straightforward; see Mehra [17], Astrom and Kallstrom [18] and Valdés-Sosa *et al* [19] for a detailed treatment.

4 Results

We choose to employ a state space model consisting of $r = 4$ AR(2) models, such that 4 major frequency bands can be described. By fitting the model to the data shown in Figure 1 these frequencies are found as 2.4Hz, 10.3Hz, 17.6Hz and 24.5Hz, corresponding to the delta, alpha, mid-range beta and low-range gamma frequency bands, respectively.

We find that the autoregressive parameters $\alpha_i^{(k)}$ in Equation (3) do not differ significantly from zero, therefore we set the AR order to $p = 0$, which helps to reduce the number of parameters to be fitted. We also impose the constraint $\beta_1^{(k)} = \cdots = \beta_q^{(k)} =: \beta^{(k)}$, such that the variances can change smoothly, and we have a further reduction of the number of parameters. Since the likelihood does not improve significantly for MA order q larger than 2, we set $q = 2$. The resulting estimate of $\beta^{(k)}$ for the 2.4Hz-component is 0.42, while for the other components values of -0.047 (10.3Hz), 0.42 (17.6Hz) and -0.25 (24.5Hz) are found.

—————————— Figure 2 about here. ——————————

In Figure 2 we show the estimated components $z_t^{(k)}, k = 1, 2, 3, 4$; these components, each corresponding to one of the four frequencies, represent a decomposition of the original data (shown again in the bottom panel of the figure), such that by summing up these components according to Equation 2, using the weights $c^{(k)}$, the original data is reproduced. Note that a pronounced increase of amplitude occurs for the 2.4Hz-component at about 10 seconds; this effect is solely obtained as a result of the maximum-likelihood model fit, without any input of prior knowledge concerning the change of the spectral composition of the data.

—————————— Figure 3 about here. ——————————

In Figure 3 we show explicitly the variances $\log\left(\sigma_t^{(k)}\right)^2$ of the components as functions of time, according to the fitted GARCH models. It can be seen that for the 2.4Hz-component the variance increases at about 8 seconds to a considerably larger value than before, and maintains that larger value within the second half of the data set. The variances of the other components do not display significant changes. This result can be readily interpreted by stating that the loss of consciousness at onset of anaesthesia is reflected almost exclusively by an increase of power in the delta band.

5 Discussion

In this paper we have proposed a new tool for quantitative description of non-stationarities in EEG time series. For this purpose we have introduced a model for decomposition of a given single-channel EEG time series into components defined by their main frequency, and we have shown how the variances of the dynamical noises driving these components can be made time-dependent by generalising the concept of GARCH modelling to the situation of state-space modelling. As a result, changes of the distribution of power over the main spectral bands of the EEG can be traced over time.

Note that by choosing the GARCH approach for describing non-stationarity of variance we obtain a method which remains suitable for real-time monitoring, in contrast to approaches which describe non-stationarities retrospectively by fitting explicitly time-dependent functions to the non-stationary parameters; the GARCH model is fully compatible with predictive modelling, since it requires only information from the past.

Once a suitable model structure has been identified (with respect to the number of components in state space and the GARCH model orders p and q) and a corresponding set of appropriate parameters has been identified through maximum-likelihood, the Kalman filter can be applied very efficiently to new data without the need of any further time-demanding computations. This enables the application of this technique to real-time monitoring of patients during surgery, e.g. it would be possible to monitor the depth of anaesthesia quantitatively by the time-varying set of variances of the relevant frequency bands. Also applications to other kinds of data arising in the neurosciences are conceivable.

EEG time series are usually recorded not from just one electrode, but from a set of electrodes covering the whole scalp; in principle, the method which we have proposed in this paper, could be applied independently to each channel of the data, but it would be desirable to have a modelling approach capable of building a single common model from all available channels simultaneously; thereby also the spatial information contained in the positions of electrodes could be incorporated. The generalisation of the method to this case will be the subject of future work.

Appendix

Here we will give the derivation of $\hat{v}^2_{t-j}$, i.e. the estimator of the noise term v^2_{t-j} in Equation (3).

Since the state prediction error w_{t-j} is not directly accessible, we derive a estimator with similar meaning. This estimator is chosen as the expectation of the product $w_{t-j}w'_{t-j}$, conditional on the data up to time $t-j$.

Let K_{t-j}, ν_{t-j} and Ω_{t-j} denote the Kalman gain, residual and inverse covariance matrix of residuals, respectively, at time $t-j$. These quantities are obtained naturally through the application of the Kalman filter (for details see Harvey *et al* (1989)). Let Var(.) and Cov(.) denote variance and covariance, respectively. Then we have

$$\begin{aligned} \hat{v}^2_{t-j} &= \mathrm{E}\left(w_{t-j}w'_{t-j}|y_{t-j}\right) \\ &= \mathrm{Var}\left(w_{t-j}|y_{t-j}\right) + \mathrm{E}\left(w_{t-j}|y_{t-j}\right)\mathrm{E}\left(w_{t-j}|y_{t-j}\right)' \end{aligned} \quad (4)$$

The expectation $\mathrm{E}\left(w_{t-j}|y_{t-j}\right)$ is equal to $K_{t-j}\nu_{t-j}$. The term $\mathrm{Var}(w_{t-j}|y_{t-j})$ represents the conditional variance of the system noise; it can be expressed as

$$\begin{aligned} & \mathrm{Var}\left(w_{t-j}|y_{t-j}\right) \\ = \; & \mathrm{Var}\left(w_{t-j}|y_{t-j-1}\right) - \mathrm{Cov}\left(w_{t-j},\nu_{t-j}|y_{t-j}\right)\mathrm{Var}\left(\nu_{t-j}|y_{t-j}\right)^{-1}\mathrm{Cov}\left(w_{t-j},\nu_{t-j}|y_{t-j}\right)' \\ = \; & Q_{t-j} - \mathrm{Cov}\left(w_{t-j},\nu_{t-j}|y_{t-j}\right)\Omega_{t-j}\mathrm{Cov}\left(w_{t-j},\nu_{t-j}|y_{t-j}\right)' \end{aligned}$$

and the covariance is obtained by

$$
\begin{aligned}
& \mathrm{Cov}\left(w_{t-j}, \nu_{t-j}|y_{t-j}\right) \\
= \; & \mathrm{E}\left(w_{t-j}\nu_{t-j}|y_{t-j}\right) - \mathrm{E}\left(w_{t-j}|y_{t-j}\right)\mathrm{E}\left(\nu_{t-j}|y_{t-j}\right) \\
= \; & \mathrm{E}\left(w_{t-j}\left(y_{t-j} - Hx_{t-j|t-j-1}\right)'|y_{t-j}\right) \\
= \; & \mathrm{E}\left(w_{t-j}\left\{\left[H\left(Fx_{t-j-1} + w_{t-j}\right) + \epsilon_{t-j}\right] - H\left(Fx_{t-j-1|t-j-1}\right)\right\}'|y_{t-j}\right) \\
= \; & \mathrm{E}\left(w_{t-j}\left\{HF\left(x_{t-j-1} - x_{t-j-1|t-j-1}\right)\right\}'|y_{t-j}\right) + \mathrm{E}\left(w_{t-j}w'_{t-j}H'|y_{t-j}\right) + \mathrm{E}\left(w_{t-j}\epsilon'_{t-j}|y_{t-j}\right) \\
= \; & Q_{t-1}H'
\end{aligned}
$$

since $\mathrm{E}\left(\nu_{t-j}|y_{t-j}\right) = 0$, $\mathrm{E}\left(w_{t-j}\left\{HF\left(x_{t-j-1} - x_{t-j-1|t-j-1}\right)\right\}'|y_{t-j}\right) = 0$, $\mathrm{E}\left(w_{t-j}w'_{t-j}H'|y_{t-j}\right) = Q_{t-1}H'$ and $\mathrm{E}\left(w_{t-j}\epsilon'_{t-j}|y_{t-j}\right) = 0$. By substituting $\mathrm{Var}(w_{t-j}|y_{t-j})$ and $\mathrm{Cov}\left(w_{t-j}, \nu_{t-j}|y_{t-j}\right)$ into Equation (4), we get

$$
\hat{v}^2_{t-j} = Q_{t-j} - Q_{t-j}H'\Omega_{t-j}HQ_{t-j} + K_{t-j}\nu_{t-j}\nu'_{t-j}K'_{t-j}
$$

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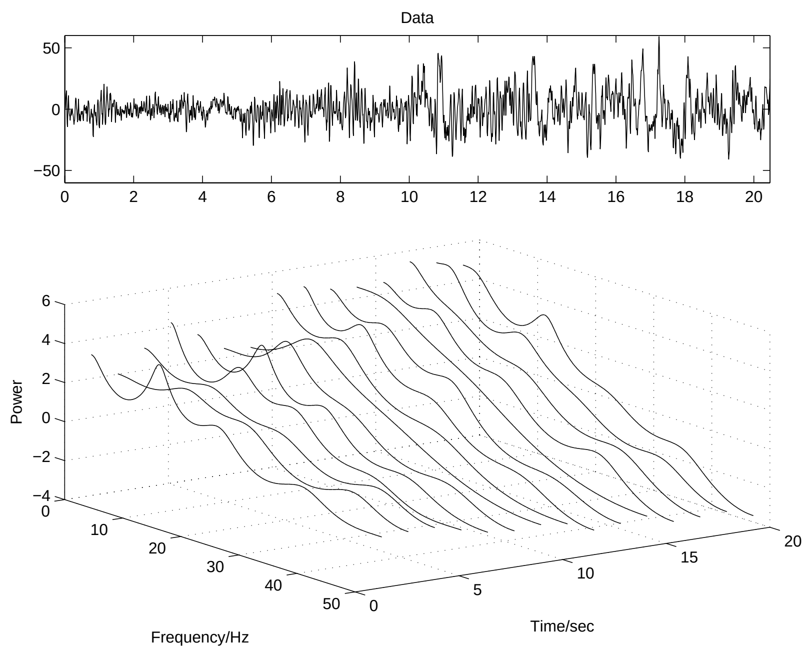


Figure 1: (top) An EEG data from the T4 electrode (versus average reference) of about 20 seconds, covers the transition from awake conscious state to anaesthesia. (bottom) A moving window spectral estimation of AR(8) models fitted to 15 segments of data, each of length 256.

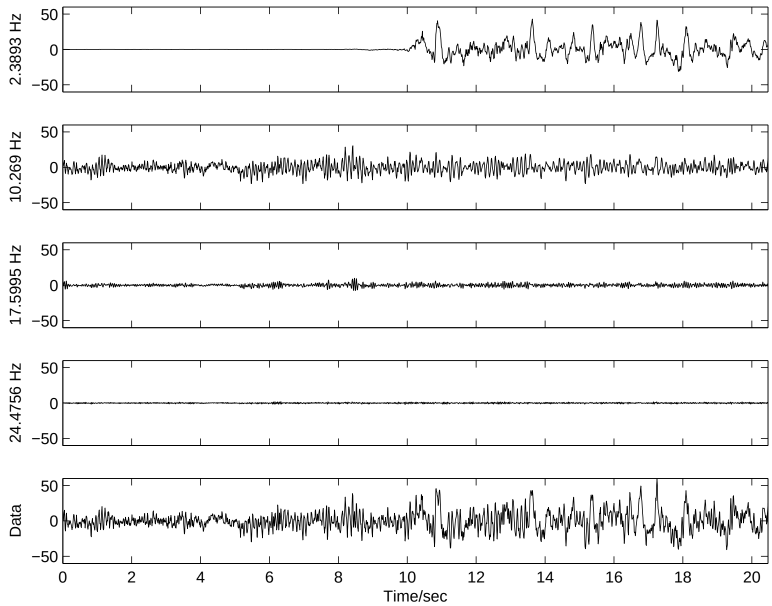


Figure 2: (top 4) Estimated source components of 4 different frequencies, scaled to the data space. (bottom) Data.

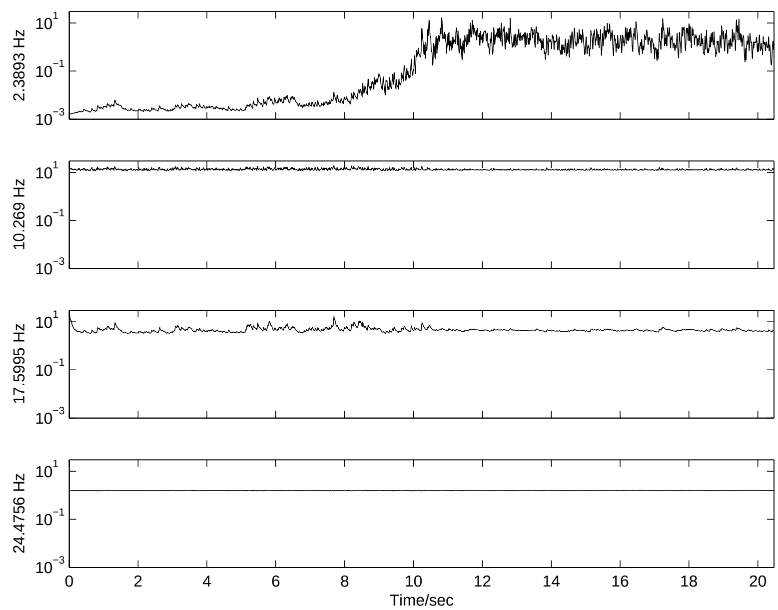


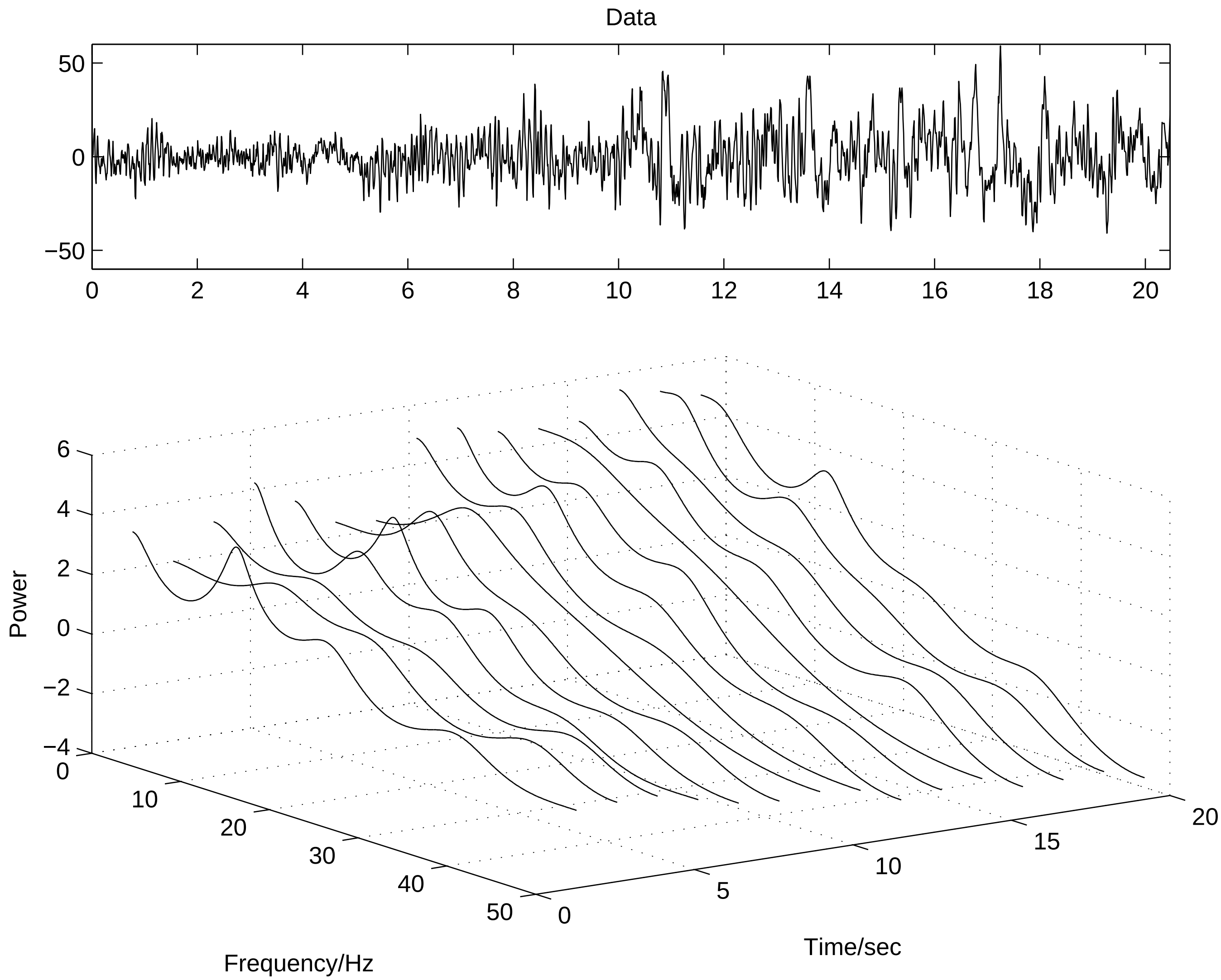
Figure 3: (top 4) Variance of prediction error of the 4 source components.

Summary

In this paper, we present a new approach to modelling non-stationarity in a time series data. We do it by a generalized state space approach. We can decomposed a given electroencephalogram (EEG) time series into a set of noise-driven processes, each corresponding to a different frequency band. We show how the variances of the dynamical noises driving these components can be made time-dependent by generalising the concept of the Generalized Autoregressive Conditional Heteroscedastic (GARCH) modelling to the situation of state-space modelling.

Nonstationarity is modelled by allowing the variances of the driving noises to change with time, depending on the state prediction error within the state space model. The method is illustrated by an application to EEG data recorded during the onset of anaesthesia.

Figure1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0
−50
0
2
4
6
8
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12
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16
18
20
Power
6
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30
40
50
Frequency/Hz
0
5
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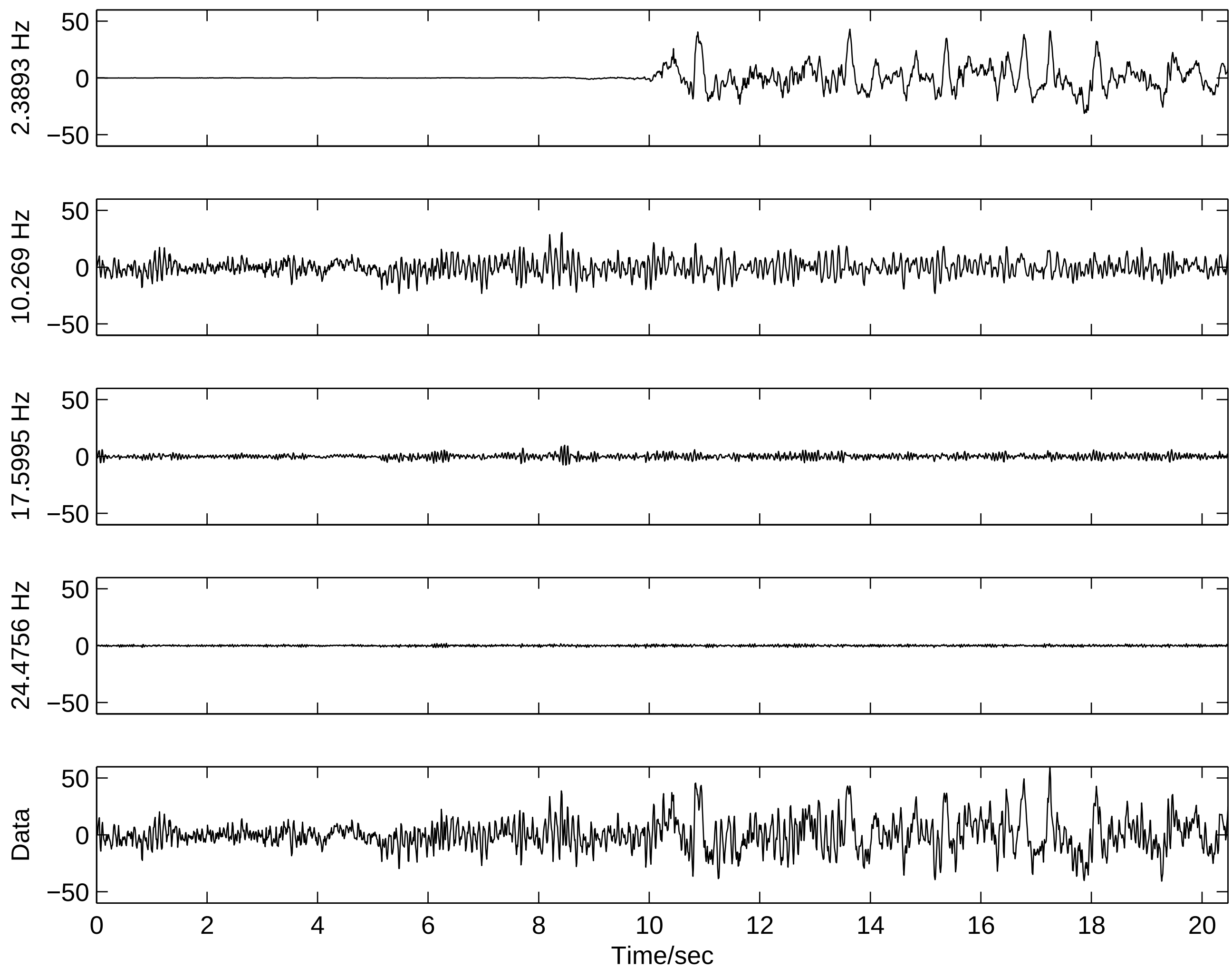
15
20
Time/sec

Figure2



2.3893 Hz
10.269 Hz
17.5995 Hz
24.4756 Hz
Data
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0
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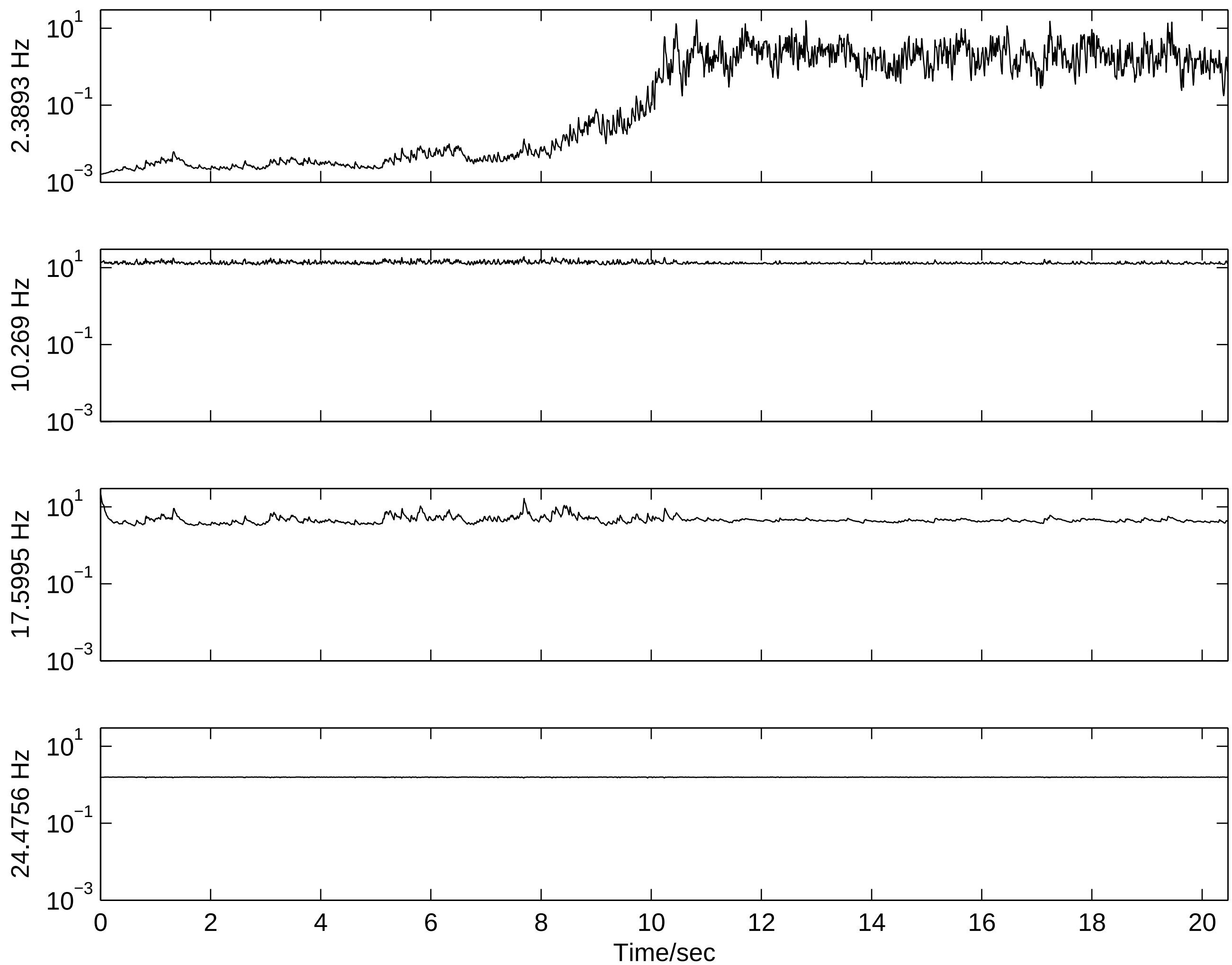
18
20
Time/sec

Figure3


2.3893 Hz
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10^{1}
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18
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Time/sec